Your Signature ______________________

Instructions:
1. For writing your answers use both sides of the paper in the answer booklet.

2. Please write your name on every page of this booklet and every additional sheet taken.

3. If you are using a Theorem/Result from class please state the result clearly and verify the hypotheses of the same.

4. **Maximum time is 3 hours and Maximum Possible Score is 100.**

Score

Q.No.	**Alloted Score**	**Score**
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

Number of Extra sheets attached to the answer script: ______________________

1. Let $\{Y_n\}_{n\geq 1}$ be a sequence of bounded random variables on $(\Omega, \mathcal{F}, \mathbb{P})$. Show that

$$Y = \liminf_{n\to\infty} Y_n$$

is measurable.

2. Let $\{X_n\}_{n\geq 1}$ be independent random variables on $(\Omega, \mathcal{F}, \mathbb{P})$. Suppose $X_n \sim \text{Normal}(0,1)$ then show that $\mathbb{P}(\limsup_{n\to\infty} \frac{X_n}{\sqrt{2\log(n)}} = 1) = 1$.

3. Let $a_n = \exp(-n)\sum_{k=0}^{n} \frac{n^k}{k!}$. Using the Central Limit Theorem evaluate $\lim_{n\to\infty} a_n$.

4. Let $\mathbb{P}, \{\mathbb{P}_n\}_{n\geq 1}$ be Probability measures on $(\mathbb{R}, \mathcal{B}_{\mathbb{R}})$. Suppose that for every subsequence $\mathbb{P}_{n_k}$ there is a further subsequence $\mathbb{P}_{n_{k_l}}$ that converges weakly to $\mathbb{P}$. Show that $\mathbb{P}_n$ converge weakly to $\mathbb{P}$.

5. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a Probability space. Let X be a random variable on it with $\mathbb{Q}$ being the distribution of X on $(\mathbb{R}, \mathcal{B}_{\mathbb{R}})$. Show that for any Borel-measurable function $f : \mathbb{R} \to \mathbb{R}$

$$\int_{\mathbb{R}} f(y) d\mathbb{Q}(y) = \int_{\Omega} f(X(\omega)) d\mathbb{P}(\omega),$$

assuming both sides exists.